Propagation and Trapping of Excitations on Percolation Clusters

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A study is presented of migration of optical or magnetic excitations on percolation clusters which terminates upon reaching a trapping site. The theory is based on the extension of results from the theory of random walks to systems without translational invariance, together with the use of scaling concepts. For the case of an excitation which resides on one type of atom in a randomly mixed crystal near the percolation threshold, new power laws for the time and concentration dependences of the mean number of sites visited at time t of the kinetics of arrival at traps are obtained. Some of these results are also tested for the first time by numerical simulations.

KEY WORDS: Diffusion; trapping; fractals; percolation.

1. INTRODUCTION

Recently there has been a growing interest in physical processes on fractal structures[1-11] which are the subject of many of the papers in this proceedings. Only a few of these problems, however, have been related to physical phenomena that can be experimentally observed.

A class of phenomena which reflects in a direct manner the fractal nature of the disordered systems in which they occur is related to the incoherent propagation of excitations on a structures characterized by a fractal geometry over a range of length scales. Examples of such systems are polymer solutions and melts, gels, epoxy resins, and mixed crystals. Consider a randomly mixed AB crystal in which an optical or magnetic excitation that resides only on type-A atoms propagates by hopping on A clusters. If the concentration of A atoms p is close to the percolation threshold, p_c the A clusters are fractal on a length scale regime bounded from above by the

correlation length $\xi \propto (p - p_c)^{-\nu}$.[7,8] The natural way to probe such diffusion microscopically is by placing randomly a third type of molecule near a small fraction of the sites of the A clusters that can register the first arrival of the excitation at these sites. Thus, the properties of the random walk probed in this manner are the number of distinct sites visited and the probability of not having arrived at any sensor site by a given time. An interesting feature of the structures that appear in this type of phenomena is that the difference between the A and B molecules may be very slight, other than the preference of one of them by the excitation. This feature leads to the possibility of creating a truly random binary system to which a percolation model applies. Such processes have been studied experimentally in mixed organic crystals.[12,13] It is assumed that the excitations migrate among the A molecules until they reach a trapping site in which they get trapped and then decay by fluorescence. The concentration of the traps is very low and they are randomly located. The kinetics of the process can be monitored by the time dependence of the fluorescence corresponding to decay through the various possible channels.

In this paper I construct a theory for the kinetics of arrival at traps of an excitation diffusing on disordered systems with fractal geometry. Most of the main results described here appeared in a recent letter.[14] This paper contains an extension of the theory of Ref. 14, as well as new numerical results which confirm some of the theoretical predictions for the first time. The model used is that of a particle or excitation performing a random walk on a random subset of allowed sites of a regular lattice. This set has a fractal structure up to a length scale ξ, and is characterized by an average translational invariance over larger length scales. A small fraction of the lattice sites are labeled randomly as traps, and the random walk terminates upon arrival at such a site. The existence of two spatial size regimes will be reflected in two time regimes with two different trapping kinetics: (a) $t < \tau$ where τ is the mean time to diffuse a distance ξ. Here the time dependence of quantities characterizing the random walk (RW) are determined by the specific fractal structure. (b) $t > \tau$. Here the time dependence of these quantities is similar to that of a RW on the embedding lattice, whereas the crossover time τ enters these relations as a parameter. The two regimes are tied together by using scaling argument related to finite size scaling. The present discussion centers on percolation clusters, but a generally similar treatment can be applied to other systems, e.g., polymers, where the propagation of excitations along the macromolecular chain has been experimentally studied.[15]

2. THE NUMBER OF SITES VISITED AND THE PROBABILITY OF SURVIVAL: GENERAL RELATIONS

The probability that a random walker on a lattice with a concentration of traps c is not yet trapped at time t after starting the walk is[15,16]

$$N(t) = \langle (1 - c)^{\tilde{S}(r_0, t)} \rangle \sim \langle e^{-c\tilde{S}(r_0, t)} \rangle \tag{1}$$

Here $\tilde{S}(r_0, t)$ is the number of distinct sites visited for a particular random walk starting at site r_0, and the bracket denote averaging over all random walks and over all initial sites r_0. Since the variance of $\tilde{S}$ for $d < 2$ is of the order of $\langle \tilde{S} \rangle^{2}$,[16] it is more convenient to treat the problem for the time regime where $c\langle \tilde{S} \rangle < 1$, so that

$$N(t) \sim 1 - c\langle \hat{S}(r_0, t) \rangle \sim e^{-cS(t)}$$
$$S(t) = \langle \tilde{S}(r_0, t) \rangle \tag{2}$$

This relation between $N(t)$ and $S(t)$ will be shown to be relevant for most cases discussed in this work. Hence $S(t)$ will be studied in detail here and in the following section.

The mean number of distinct sites visited for an infinite ordered lattice is given by the inverse Laplace transform of $S(\omega)$ with[16,17]

$$S(\omega) = \frac{1}{\omega^2 P_0(\omega)} \tag{3}$$

Here $P_0(\omega)$ is the Laplace transform of the probability to return to the origin after time t. For a random walk on a fractal set translational invariance on which the derivation of Eq. (3) is based no longer exists and Eq. (3) will be replaced by a more general version which will now be derived.

The mean number of distinct sites visited by a random walk starting at the origin r_0 is generally given by[17]

$$S(r_0, t) = 1 + \sum_r \int_0^t f(r_0, r, T)\, dT \tag{4}$$

Here $f(r_0, r, T)$ is the probability that the random walker would reach site r at time T for the first time. This quantity can be related to the probability of the random walker to be at site r at time t after originating from r_0, $P(r_0, r, t)$, and the local probability to be at r at time t, given that it was at r at time $t' < t$, $P(r, r, t - t')$ by the integral relation

$$P(r_0, r, t) = \delta(t)\,\delta(r - r_0) + \int_0^t f(r_0, r, T)\, P(r, r, t - T)\, dT \tag{5}$$

One can now Laplace transform this relation and solve for $f(r_0, r, \omega)$:

$$f(r_0, r, \omega) = \frac{P(r_0, r, \omega) - \delta(r - r_0)}{P(r, r, \omega)} \tag{6}$$

Using this result together with the Laplace transformed version of Eq. (4) one obtains

$$S(\omega) = \frac{1}{\omega} \left\langle \sum_r \frac{P(r_0, r, \omega)}{P(r, r, \omega)} \right\rangle_{r_0} \tag{7}$$

Here $P(r_1, r_2, \omega)$ is the Green's function for diffusion, and the averaging is over r_0. Equation (4) implies that $S(\omega)$ cannot always be expressed as a simple extension of Eq. (3) in which $P_0(\omega)$ is replaced by an averaged $\langle P_0(r_0, r_0, \omega) \rangle$.

3. THE NUMBER OF SITE VISITED ON PERCOLATION CLUSTERS

For the problem of percolation clusters at $p > p_c$, one has two distinct cases according to whether the random walk starts on an infinite cluster, or on any allowed site.

For the case in which the excitation starts on the percolating cluster for $p = p_c$, one can assume that after sufficiently long time, for the asymptotic time dependence of the probability to return to the origin does not depend on the particular starting point. Thus for small ω, $P(r, r, \omega) \propto g(r)\omega^x$ for all r on the percolating cluster. The amplitude $g(r)$ should be bounded and nonzero, since the infinite cluster is a connected set of points. Using this asymptotic from in Eq. (7), leads to a recovery of Eq. (3) for sufficiently small ω. In order to obtain an expression for $P_0(\omega)$ on the percolating cluster the following scaling argument is used:

$$P_0(t, p - p_c) \sim \left(\frac{t}{\tau}\right)^{-(d/2)} \frac{1}{\xi^d \Pi(p)} f\left(\frac{t}{\tau}\right) \tag{8}$$

$$f(x) \begin{array}{ll} \to 1, & x \gg 1 \\ \to x^\sigma, & x \ll 1 \end{array}$$

Here τ is the mean time to diffuse a distance ξ, $\tau = \xi^2/\Delta(p)$, $\Delta(p) \sim (p - p_c)^{\mu - \beta}$ is the diffusion coefficient on the percolating cluster and μ is the percolation conductivity exponent.[18,19] The justification for this scaling is twofold:

(a) At $t > \tau$ the time-dependent behavior of the random walk will be similar to that of a random walk on a d-dimensional lattice with a renor-

malized unit cell of size ξ. The number of steps of the RW is given by the renormalized time t/τ, and $P_0(t)$ is normalized by the number of available sites in the unit cell $\xi^d \Pi(p) = \xi^D$. Here $\Pi(p)$ is the probability to be on the infinite cluster $\Pi(p) \propto (p - p_c)^\beta$ and D is the fractal dimensionality of percolating clusters, $D = d - \beta/v$.[7b]

(b) At $t < \tau$ or $p = p_c$ one expects no dependence of $P_0(t)$ on $p - p_c$. The second condition leads to $\sigma = d/2 - Dv/(\mu + 2v - \beta)$ and the following time dependence of $P_0(t)$ is obtained:

$$P_0(t) \sim t^{-(\tilde{d}/2)}$$

$$\tilde{d} = \frac{2Dv}{\mu + 2v - \beta} \tag{9}$$

The anomalous properties of the random walk on a percolating cluster reflected in this time dependence of $P_0(t)$ were first discussed in the context of the anomalous time dependence of the mean square displacement from the origin: $R^2(t) \propto t^{(\tilde{d}/D)}$.[7a,8,19] A result identical to Eq. (9) was first derived by arguments relying on this time dependence.[2,3,5] The parameter $\tilde{d}$ was named the fracton dimensionality of the percolating cluster in Ref. 2, where it has been conjectured that for all dimensionalities $\tilde{d} = 4/3$. Since $\tilde{d} < 2$ from Eq. (3)[15,16]

$$S(t, p_c) \propto P_0(t)^{-1} \sim t^{(\tilde{d}/2)} \tag{10}$$

Thus on the percolating cluster at $p = p_c$ the random walker visits on the average all the sites in the diffusion volume, i.e., the exploration of sites is compact.[20]

In order to obtain $S(t, p)$ for $p > p_c$ a similar scaling argument is used. $S(t, p)$ can be expressed as follows:

$$S(t, p) = \tau^r t \phi(t/\tau)$$

$$\phi(x) \begin{array}{l} \to 1, \quad x \gg 1 \\ \to x^r, \quad x \ll 1 \end{array} \tag{11}$$

This scaling relation expresses the following two conditions: (i) At long times $t > \tau$ the number of sites visited depends on t in the same manner as it would for a random walk on a regular lattice. Thus for $d = 3$, $S(t, p) \propto t$. (ii) For $t < \tau$, $S(t, p) \propto t^{(\tilde{d}/2)}$ and does not depend on $p - p_c$. The later condition leads to $r = \tilde{d}/2 - 1$. Using Eq. (9) we get

$$S(t, p) = E(p - p_c)t$$

$$E(p - p_c) \propto (p - p_c)^{\mu + (2-d)v} \tag{12}$$

In the typical excitation propagation and trapping experiment the excitation is created optically.[12,13] Thus the excitation is not constrained to start on the percolating cluster and can start on any A cluster. Next, $S(t, p)$ is calculated for this case. For convenience I first consider the case $p = p_c$. The expression for $S(t, p)$ is no given by

$$S(t) = \sum_n \rho(n)\, S(n, t) \tag{13}$$

Here $\rho(n)$ is the probability that the excitation starts on a cluster of n sites and $S(t, n)$ is the number of sites visited on such a cluster. Note that the dependence on r_0 inside the cluster is neglected. $\rho(n)$ is related to the distribution of clusters of n sites at $p = p_c$, $\pi(n) \sim n^{-(d/D+1)}$ by $\rho(n) = n\pi(n)$.[21] A related averaging procedure was carried out in Ref. 7a for the mean square displacement of a random walk starting from any cluster. For a given t the form of $S(n, t)$ depends on n:

$$S(n, t) \propto \begin{cases} t^{\tilde{d}/2} & t^{\tilde{d}/2} < n \\ n & t^{\tilde{d}/2} > n \end{cases} \tag{14}$$

From Eqs. (12)–(14) a new power law dependence for $S(t)$ is obtained:

$$S(t) \sim t^{\hat{d}/2}$$
$$\hat{d} = \tilde{d}(2 - d/D) \tag{15}$$

It is interesting to note that $\hat{d} < \tilde{d}$ $(d > D$ for all $d)$, so that the exploration is slower than in the case where the diffusion is limited to the percolation cluster. Also in contrast to the fracton dimensionality of the percolating cluster, $\hat{d}$ is dimensionality dependent and has the mean field $(d = 6)$ value of $2/3$.

The form of averaging over clusters expressed by Eq. (13) is intutitively clear. Note that this results could also be reached by starting from Eq. (4) and using the following relations: (a) $P(r_0, r, \omega) = 0$ if r_0 and r are not on the same cluster. (b) $\sum_r P(r_0, r, \omega) = 1/\omega$ where the summation is over all sites in the cluster to which r_0 belongs. (c) $P(r, r, \omega) \sim \omega^{\tilde{d}/2 - 1}$ for $\omega > n^{-2/\tilde{d}}$, and $P(r, r, \omega) \sim 1/n$ for $\omega < n^{-2/\tilde{d}}$, when r is on a cluster of n sites. (d) The probability that r_0 is on a cluster of n sites is proportional to $\rho(n)$.

Note that it is incorrect to use Eq. (3) with an average $\langle P_0(\omega)\rangle$. By using similar derivation to that leading to Eq. (12) one obtains $\langle P_0(t)\rangle \propto t^{(-\tilde{d}d/2D)}$ for the case where the excitation starts on any cluster. Recent numerical results confirm this time dependence.[22] The correct way to average becomes even less obvious for the general problem in which the diffusion rate on the B clusters is finite but very small. Then one has to resort to Eq. (7) as a starting point.

4. THE SURVIVAL OF EXCITATIONS IN THE PRESENCE OF TRAPS AT THE PERCOLATION THRESHOLD

In the short time regime such that $cS(t, p_c) < 1$, the approximation expressed by Eq. (2) is valid. Thus, the number of survived excitations is given by Eq. (2) together with Eq. (12) for $S(t, p_c)$. A more complicated situation prevails at longer times. In this case, fluctuations in $\tilde{S}(t)$, which are strong since $\tilde{d} < 2$, become important. At very long times $N(t)$ is dominated by the contributions of random walks characterized by unusually low values of $\tilde{S}(t)$, and the statistics of rare events becomes relevant. For disordered fractals such as the percolating cluster, fluctuations of the local geometry from the average properties described by D and $\tilde{d}$ may also become important.

A lower bound for $N(t)$ at large t can be derived by the following argument: The probability that a diffusing excitation has not reached the boundary of a region of size R by time t is given by $\exp[-E(R)t]$, where $E(R)$ is the lowest eigenvalue of the diffusion equation in a region of size R. On a fractal with fracton dimensionality $\tilde{d}$: $E(R) \sim R^{-(2D/\tilde{d})}$. The probability that this region is free of traps is proportional to $\exp(-cR^D)$. The lower bound is now given by the integral:

$$N(t) > \int_0^t e^{-E(R)t} e^{-cR^D} dR \tag{16}$$

Using this result and estimating the integral by an asymptotic approximation leads to

$$N(t) \sim \exp[-Ac^{(2/\tilde{d}+2)} t^{(\tilde{d}/\tilde{d}+2)}] \tag{17}$$

Here A is a constant of order unity. This result is an extension to a fractal system of recent results for the long-time behavior of $N(t)$ on regular lattices.[23] It has been proved rigorously that Eq. (17) is also an upper bound to $N(t)$ for regular lattices at all dimensionalities,[24,25] so that this expression is exact asymptotically at long times. One certainly expects Eq. (17) to be an exact asymptotic result for fractal systems, where $\tilde{d} < 2$, since here random walks are recurrent, and all of the sites in the diffusion volume are visited with probability unity. The following argument is not a rigorous proof but it takes into account the recurrence property: The random walks that dominate the quantity $\langle \exp[-c\tilde{S}(t)] \rangle$ at very long time are those spending an unusually long time in visiting a relatively small number of distinct sites. Consider the probability distribution of $\tilde{S}(t)$ conditioned on the random walk not having exitted a sphere of radius R in time t: $P(\tilde{S} | R(t) < R)$. Owing to the recurrence property one expects this

probability distribution function to be peaked near R'^D in the limit of very long times, where R' is smaller than but very close to R, and where R is kept fixed. In the limit $\tilde{S}(t) < t^{\tilde{d}/2}$, $\tilde{S}(t)$ will be have a probability distribution:

$$P(\tilde{S}(t)) = \int P(\tilde{S} \mid R(t) < R) P_E(R(t) < R) \, dR \sim e^{-t\tilde{S}^{-2/\tilde{d}}} \tag{18}$$

Here $P_E(R(t) < R)$ is the probability that the random walk has not exitted the sphere of radius R, $P_E \propto \exp[-E(R)t]$. Using this result for evaluating the quantity $\langle \exp[-c\tilde{S}(t)] \rangle$ one obtains in the long-time limit $N(t) \propto \exp[cS(t)]^{(2/2+\tilde{d})}$, which agrees with Eq. (17).

Combining this result with the expression for $N(t)$ for short times leads to the following scaling form for $N(t)$:

$$N(t) \propto e^{-cS(t)\,h(cS(t))}$$

$$h(x) = \begin{cases} 1 & x \ll 1 \\ x^{-\tilde{d}/(\tilde{d}+2)}, & x \gg 1 \end{cases} \tag{19}$$

This scaling form is confirmed by numerical results reported in this proceeding,[26] which show that the first several moments of the distribution function of $\tilde{S}(t)$ are proportional to the corresponding powers of $S(t)$, and by the numerical results discussed below.

5. NUMERICAL RESULTS

In order to test these predictions simulations of diffusion on percolating clusters in the presence of traps were carried out. A large percolating cluster at $p = p_c$ was generated on a $(150)^2$ square lattice and a fraction c of the sites were randomly designated as traps. Each random walk starts at the origin and terminates upon reaching a trap. At each time step the number of survived random walks is recorded. The total number of random walks samples for each concentration of traps was $\sim 1.5 - 2 \times 10^6$, while a new percolating cluster was generated for each group of 100 random walks. The large number of random walks sampled was chosen so that the number of survived random walks at the longest time studied was several hundreds, to assure satisfactory statistics in this rare events regime. The results for $-\log[N(t)/N(0)]$ vs. the scaling variable $(c^{(2/\tilde{d})}t)$ are plotted in Fig. 1 on a log–log scale. The values of c represented in the plot are $c = 0.02$, 0.05, 0.075, 0.1.

The results show that the scaling relations expressed in Eq. (19) are obeyed. The collapse of the results for different values of c onto a single curve was achieved with a choice of $\tilde{d} = 0.655$. The results clearly show that
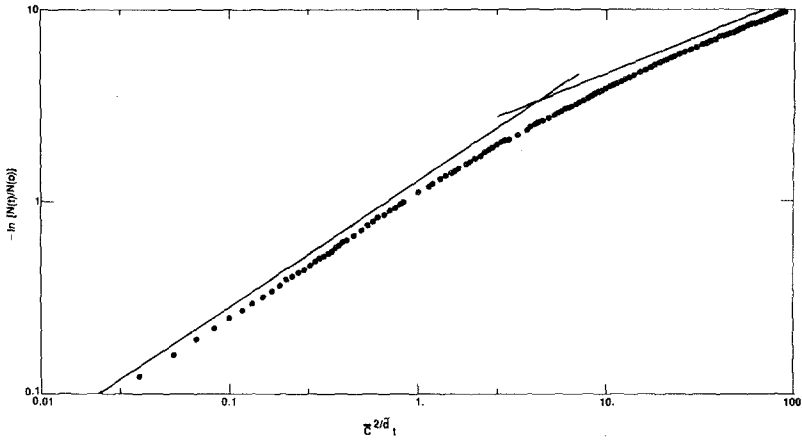
Fig. 1. The logarithm of $-\log[N(t)/N(0)]$ vs. the logarithm of $\tilde{c}^{2/\tilde{d}}t$ where $\tilde{c} = -\log(1-c)$, for values of trap concentration $c = 0.02, 0.05, 0.075, 0.1$. The results for various c values are all represented by the solid circles. The value of $\tilde{d}$ used to obtain the scaling fit is $\tilde{d} = 0.655$. The slopes of the two straight lines are 0.66 and 0.4.

$N(t)$ undergoes a slow crossover from a $\exp(-ct^{(\tilde{d}/2)})$ behavior at short times to a behavior consistent with Eq. (17) and Eq. (19) at long times. These are the first results which demonstrate this long-time behavior numerically.

It would be very interesting to study the survival probability at even longer time than studied here. Percolation clusters are disordered fractals, and one can expect fluctuations in their structure to come eventually into play. Thus, the traps reached at very long times are likely to be situated on difficult to reach quasi-one-dimensional regions such as long stringy chains branching off the backbone of the cluster. It seems plausible that at very long times the decay law for $N(t)$ will be even slower than Eq. (17), reflecting the one-dimensional nature of the search for such out-of-the-way traps. In this limit a $\log[N(t)] \propto -c^{(2/3)}t^{(1/3)}$ decay corresponding to Eq. (17) in one dimension, can be expected.

6. ON THE PERCOLATING CLUSTER ABOVE P_c

For the case in which the excitation starts the diffusion on the percolating cluster above p_c, the interplay between two characteristic times determines the nature of the trapping process: (1) the time to diffuse a distance ξ, $\tau \sim (p - p_c)^{(\mu + 2\upsilon - \beta)}$, the upper limit of the fractal diffusion regime; (2) the mean time to reach a trap at $p = p_c$, τ_s for which $c\tau_s^{(\tilde{d}/2)} = 1$, which appeared implicitly in the previous section. For times such that $t > \tau$ the behavior of $N(t)$ will be the similar to the case of diffusion on a d-

dimensional regular lattice. If the dimensionality of the lattice is $d = 3$, then the expression $N(t) \propto \exp[-cS(t, p)]$, where $S(t, p)$ is given by Eq. (12), should be a very good approximation even at long times such that most of the excitations are trapped, since in $d = 3$ the fluctuations in $\tilde{S}$ are small. This point is borne out by recent numerical studies of averages such as these appearing in Eq. (1)[27a] as well as by an analytical estimate of the corrections to the approximation $\langle \exp(-cS) \rangle = \exp(-c\langle S \rangle)$.[27b] For $\tau \gg \tau_s$, i.e., very close to p_c and/or for large trap concentration the behavior of $N(t)$ up to time τ will be similar to that at p_c discussed in the previous two sections. For low trap concentrations (which corresponds to the situation in the experiments on mixed organic crystals) except for very close to p_c, $\tau_s > \tau$. In this case most of the trapping will occur during diffusion over distances greater than ξ. In $d = 3$ the short-time regime where $t < \tau_s$ ($ct^{(\tilde{d}/2)} < 1$), and the nonfractal regime $t > \tau$ overlap for $p - p_c > 0.05$ for $c \leqslant 10^{-3}$. Thus for small trap concentrations the approximation using the mean number of sites visited for $\log[N(t)/N(0)]$ is expected to be good for all times except very close to p_c.

7. THE SURVIVAL OF EXCITATIONS STARTING ON ANY CLUSTER

For the general case in which the excitation may start to diffuse from an origin located on a cluster of any size, the number of untrapped excitation is given by

$$N(t) = \sum_n \rho(n) N(n, t) \tag{20}$$

Here $N(n, t)$ is the number of untrapped excitations on a cluster of n sites at time t:

$$N(n, t) = e^{-cS(n,t)} \tag{21}$$

where $S(n, t)$ is given by Eq. (14). Note that for long times $S(n, t) \propto n$ for all realizations of the random walk so that Eq. (21) becomes exact. For long times the dominant contribution to $N(t)$ in Eq. (20) comes from finite clusters that are void of traps. Thus, even close to the percolation threshold, Eq. (20) with $N(n, t)$ given by Eq. (14) should be a very good approximation.

At $p = p_c$, in the time regime defined by $ct^{(\tilde{d}/2)} < 1$, i.e., $t < \tau_s$, $N(t)$ will be given by substituting $S(t)$ of Eq. (15) in Eq. (2). At longer times $N(t)$ will cross over to a time-independent limit given by

$$N(t) \to 1 - c^{(d-D)/D}, \qquad ct^{\tilde{d}/2} \gg 1 \tag{22}$$

For $p > p_c$

$$N(t) = \Pi(p)e^{-cS(t,p)} + \lceil 1 - \Pi(p)\rceil N_f(t, p)$$

$$N_f(t, p) = \sum_n \rho(n)e^{-cS(n,t)} \tag{23}$$

Here $N_f(t, p)$ is the number of untrapped excitations that were initially created on finite clusters. The summation over n has an upper limit of ξ^D, since the finite clusters for $p > p_c$ are bounded from above in size by ξ.[28] $N_f(t, p)$ has several different limiting behaviors in different time and p regimes:

(a) $t < \tau$: $N_f(t, p)$ is given by Eq. (2) with $S(t)$ of Eq. (15) for $ct^{(\tilde{d}/2)} < 1$, and crosses over to Eq. (22) for $ct^{(\tilde{d}/2)} > 1$, i.e., a behavior similar to that of $N(t)$ at $p = p_c$.

(b) $t > \tau$: Here $N_f(t, p)$ does not depend on t. Sufficiently close to p_c, so that $c\tau^{(\tilde{d}/2)} = c\xi^D > 1$, N_f is again given by Eq. (22). Further away from p_c, for $c\xi^D < 1$,

$$N_f \sim 1 - c(p - p_c)^{v(d-2D)}/[1 - \Pi(p)] \tag{24}$$

8. SUMMARY

I have described results relevant for phenomena controlled by diffusion and trapping on fractals. The discussion centered on percolation clusters, but the treatment can be extended to other systems, e.g., to polymers.

A direct relation between the number of untrapped excitations and the mean number of sites visited in time t on various clusters exists in most cases relevant to experiment. In the case where the diffusion takes place on a fractal up to long times, i.e., at $p = p_c$ on the infinite cluster, this direct relationship is accurate only in a time regime by the end of which a fraction of ~ 0.1–0.2 of the excitations are not yet trapped. At long times the decay is slower than $\exp[-cS(t)]$ and is given by Eq. (19). This form of decay agrees very well with numerical results.

The matching of the model to experimental systems is less certain at long times. In this case the hopping of the excitation between atoms that are not nearest neighbors might become significant. The nearest-neighbor models used here can be extended to include hopping to further sites. Another direction in which the present work can be extended is to use a more detailed description of the topology percolating clusters, such as taking into account the different local fractal dimensionalities of the backbone and of the dead ends, and the fluctuations in local geometry.

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